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Late D-term inflation and the cosmological moduli problem in TeV scale strings

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Abstract: We show that a short period of late D-term inflation can solve the cosmological moduli (radion) problem of (asymmetric) inflation at the TeV scale. Late inflation happens after the large compact dimensions are stabilized which is crucial for obtaining the extremely small Hubble constant and inflaton mass required.

Keywords: Cosmology of Theories beyond the SM, Physics of the Early Universe, Compactification and String Models.

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1. Introduction

A generic problem in late cosmology of supersymmetric theories with exact flat directions (or moduli) is the cosmological moduli problem [1]. An important example is superstring theory in which there are many moduli such as the dilaton, the untwisted and twisted moduli which parametrize different string vacua. Another example is radii of compact dimensions in gravitational theories with more than four dimensions. (Actualy these are related to the dilaton and untwisted moduli of string theory.) These moduli are massless to all orders in perturbation theory and obtain masses only by nonperturbative effects which break supersymmetry. These effects lift the flat directions and the moduli get a potential with a minimum. As a result, the moduli start to roll down from their initial value and begin to oscillate about their minimum. Since the initial value of the moduli fields are arbitrary, there may be a large energy density stored in the moduli fields. The energy density stored in radiation decreases as $a(t)^{-4}$ compared to $a(t)^{-3}$ for moduli ($a(t)$ is the scale factor of the universe), after some time the moduli energy density dominates the universe. This is unacceptable since it will modify the successful predictions of nucleosynthesis and later stages of Big Bang cosmology.

The cosmological moduli problem persists in models of inflationary cosmology. The only difference is that after inflation and reheating of the universe, the moduli start rolling down to their true minimum from their minimum during inflation instead of an arbitrary initial value. Thus, even though immediately after inflation the universe is radiation dominated after some time there is a danger that the moduli energy density will take over. An elegant solution to the cosmological moduli problem is late inflation, i.e. a second stage of inflation that occurs after the moduli start to oscillate about their minimum [2]. (For another solution to the moduli problem see ref. [3].) If late inflation results in N e-foldings then the moduli energy density is diluted by a factor of e^{-3N} so that for large enough N the problem is solved. In supersymmetric theories the moduli masses are of $O(\mathrm{TeV})$ and therefore one needs inflation around or below this scale.

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Recently, a new scenario with (two or more) large internal dimensions and TeV scale strings has been proposed [4, 5]. In this scenario our world lives on a brane on which there are matter and gauge interactions whereas gravity lives only in the bulk. The weakness of gravity with respect to the gauge interactions is a result of the large internal dimensions. It is clear that this scenario requires a revision of cosmology above the nucleosynthesis scale, $\sim$ MeV. In particular, inflation has to be realized in a new way. There have been a number of works related to cosmology and inflation in the presence of large compact dimensions [6]–[17] and of branes embedded in higher dimensional spaces [18]. It is easy to see that the cosmological moduli problem is even more severe in these new inflationary models due to the very low energies involved. In this letter, we consider the case of two large dimensions with a string scale (or six dimensional Planck scale) of $M_s \sim 50$ TeV [20]. However, all our results can be easily generalized to the cases with more than two large dimensions.

In [8], it was shown that in asymmetric inflation there is a radion problem. The radii of the compact dimensions have very small masses of $O(10^{-9}$ GeV) and their energy density can dominate the universe at late times. Similar problems arise in other inflationary models at the TeV scale (such as D-term inflation [17] or brane inflation [12]) which generically have moduli with similar masses. The moduli problem can be solved by late inflation as mentioned above but now one needs inflation with a Hubble constant of at least $H < 10^{-9}$ GeV. (In fact, we will see that there is a much stronger bound on H.) Moreover, the inflaton mass must be smaller than H for inflation to occur and this is difficult to achieve. With such a light inflaton it is very hard to reheat the universe to above the nucleosynthesis scale.

In this letter, we show that late D-term inflation [19] can provide a solution to the cosmological moduli (or radion) problem. Since the Hubble constant is very small late D-term inflation occurs after the large compact dimensions are stabilized. Then the large dimensions give rise to the extremely small gauge coupling (for the anomalous U(1)) which is crucial for obtaining the extremely small Hubble constant and inflaton mass required. In the next section we show how late D-term inflation solves the radion problem of early asymmetric inflation. Section 3 contains our conclusions and a discussion of our results.

2. The radion problem in early asymmetric inflation

In this section, we assume that the horizon, monopole and flatness problems are solved by an early phase of asymmetric inflation at a Hubble constant $H \sim M_s \sim 5 \times 10^4$ GeV in a theory with TeV scale gravity [8]. (The scale 5×10^4 GeV is the lowest quantum gravity scale possible in light of astrophysical bounds. [20].) Asymmetric inflation occurs when the size of all the compact dimensions are of $O(M_s^{-1})$. During asymmetric inflation the noncompact dimensions inflate whereas the compact ones are slowly changing. In [8] it was shown that enough e-foldings can be obtained with (almost) scale invariant density perturbations of the magnitude COBE data requires. After asymmetric inflation ends the large compact dimensions stabilize at their large radii. As noted in [8], in this scenario there is a cosmological moduli problem associated with the radii of the compact dimensions, or for simplicity the radion r. After early inflation ends and the large internal dimensions stabilize the mass of the radion on general grounds (since it is a modulus) becomes $m_r \sim M_s^2/M_P \sim 3\times10^{-9}$ GeV. In addition, the universe is reheated by inflaton decay into gravitons up to a temperature $T_R \sim 10$ MeV which is high enough for nucleosynthesis.

The generic cosmological moduli problem manifests itself as the radion problem in this scenario. Following the inflationary era and reheating of the universe there is a danger that the radion energy density will dominate the radiation energy density after a period of time since the former decreases much more slowly than the latter. As usual the simplest way to solve this problem is to have a second phase of late inflation in order to dilute the radion energy density. It turns out that in this context a late inflationary period with about 6-7 e-foldings can solve the radion problem [8].

Since the radion mass is small i.e. $m_r \sim 3\times10^{-9}$ GeV the late inflation required to dilute the radion energy density has to happen at a very small Hubble constant $H < m_r$. In fact, there is a stronger bound on H coming from the requirement that the late inflationary period should not reintroduce the radion problem [13]. If H during late inflation is too large then the minimum of the radion potential will be displaced by a large amount from its true value (obtained after late inflation) and then there will again be too much energy stored in the oscillations of the radion field. For two large dimensions the minimum during late inflation is given by ($r_i = 1$ is the minimum after late inflation)

$$r_i \sim 1 + \frac{2H^2}{4m_r^2}\,. \tag{2.1}$$

On the other hand, the energy stored in the radion field is

$$V_r = 2m_r^2 M_P^2 (r_i - 1)^2\,. \tag{2.2}$$

Demanding that the radion energy density does not overclose the universe today requires that V_r sholud satisfy

$$\frac{V_r}{T_R^3} < 1.5\times10^{-9}\ \mathrm{GeV}\,. \tag{2.3}$$

As a result, the Hubble constant during late inflation has to satisfy [13]

$$H < 4\times10^{-19}\ \mathrm{GeV}\left(\frac{T_{RH}}{10\ \mathrm{MeV}}\right)\left(\frac{m_r}{10^{-2}\ eV}\right), \tag{2.4}$$

which is a constraint many orders of magnitude stronger than $H < m_r$.

In order to have late inflation the inflaton mass has to be smaller than this value of the Hubble constant, i.e. $m_\phi < H$ resulting in a very small inflaton mass. It is generically very difficult to obtain such small inflaton masses. Moreover, since late inflation reduces the original reheating temperature obtained after early inflation by an exponential factor, the decay of this light inflaton must reheat the universe. For such small inflaton masses it is very difficult to reheat the universe back to $T_R \sim 10$ MeV.

In this letter, we show that late inflation can be realized as D-term inflation in four dimensional compactifications of type I string theory with D-branes (or type IIB string orientifold compactifications) [21, 22]. We assume that the compact radii are stabilized after early asymmetric inflation but before the late D-term inflation. This is crucial for the scenario because as we will see the very small gauge coupling required for the anomalous U(1) during late D-term inflation is a direct result of the large compact dimensions [17]. For concreteness we consider a case of two D5 branes intersecting over three noncompact dimensions which describe our world. One of the D5 branes is wrapped over a small two torus of area M_s^{-2} whereas the other is wrapped over a large torus of area $R^2 \sim M_P^2/M_s^4 \sim 2 \times 10^{17}$ GeV^{-2}. In this case, the four dimensional gauge coupling corresponding to the interactions on the brane wrapping the large torus is

$$g_4^2 = \frac{g_6^2}{R^2 M_s^2} \,. \tag{2.5}$$

Assuming $g_6 \sim 1$ we get $g_4^2 \sim 3 \times 10^{-27}$. The type I orbifold models generically have an anomalous U(1) gauge symmetry with an anomalous D-term [21, 22, 23]. We assume that the anomalous U(1) arises from the gauge interactions on the above D5 brane with a tiny coupling. Then there is a D-term contribution to the scalar potential

$$V_D = g_4^2 \left| -|\sigma|^2 + M^2 \right|^2 \,. \tag{2.6}$$

Here σ is the trigger field which carries -1 charge under the anomalous U(1) and M is the twisted modulus. The VEV of the twisted modulus (which parametrizes the blowup of the orbifold singularity) M fixes the vacuum energy during inflation or the Hubble constant H. In addition we assume a tree level superpotential between the trigger field and the inflaton ϕ

$$W = \lambda \phi \sigma^2 \,. \tag{2.7}$$

Such tree level superpotentials are also generic to type I orbifold models [22]. Note that the inflaton is neutral under the anomalous U(1). We assume that the inflaton ϕ lives on the second D5 brane wrapped over the small torus and therefore the Yukawa coupling $\lambda \sim 1$. The superpotential gives rise to the scalar potential

$$V_F = \lambda^2 \phi^2 \sigma^2 + \lambda^2 \sigma^4 \,. \tag{2.8}$$

The total scalar potential $V_{tot} = V_D + V_F$ gives rise to D-term inflation (which is a type of hybrid inflation [24]).

The total scalar potential has two minima; one at $\sigma = M$ and $\phi = 0$ and the other at $\sigma = 0$ and ϕ free. Inflation occurs for the initial conditions of large ϕ and small or vanishing σ. Then the vacuum energy is dominated by the D-term and

$$V_0 \sim H^2 M_P^2 \sim g^2 M^4 \,. \tag{2.9}$$

The masses of the two scalars are given by

$$m_\sigma^2 = \lambda^2 \phi^2 - 2g^2 M^2 \tag{2.10}$$

and

$$m_\phi^2 = \lambda^2 \sigma^2 \,. \tag{2.11}$$

For $\phi > \sqrt{2} gM/\lambda = \phi_{crit}$, m_σ^2 is positive and larger than the Hubble constant, H. As a result, σ rolls down to its minimum at $\sigma = 0$ very fast. At the minimum the tree level inflaton mass vanishes. However, there is a one-loop contribution to the inflaton mass since the nonzero vacuum energy breaks supersymmetry

$$V_1 = g^2 M^4 \left(1 + \frac{g^2}{16\pi^2} \log \frac{\lambda^2 \phi^2}{\mu^2}\right) . \tag{2.12}$$

The inflaton mass becomes

$$m_\phi^2 = \frac{g^4 M^4}{16\pi^2 \phi^2} \,. \tag{2.13}$$

Because of this small mass the inflaton slowly rolls down its potential until $\phi \sim H$ and then inflation ends. The inflaton rolls down to its minimum at $\phi = 0$. On the other hand, m_σ^2 becomes negative and σ rolls down to its new minimum at $\sigma = M$. The vacuum energy vanishes and supersymmetry is restored.

For late D-term inflation we take the anomalous D-term scale to be the string scale, i.e. $M \sim M_s \sim 5 \times 10^4$ GeV. Then the Hubble constant during late inflation is $H \sim g_4 M^2/M_P \sim 10^{-22}$ GeV which is small enough to satisfy eq. (2.4). As noted earlier we need about 6-7 e-foldings to solve the radion problem. The number of e-folds during D-term inflation is given by

$$N \sim \left(\frac{\phi}{M_P}\right)^2 \frac{4\pi^2}{g^2} \tag{2.14}$$

For $N \sim 6$ we need the inflaton VEV to be $\phi \sim 10^4$ GeV $\sim M_s$ which is its natural value. For N e-foldings we find that the inflaton mass is from eqs. (2.13) and (2.14) $m_\phi^2 \sim g^2 M^4/4NM_P^2 \sim H^2/4N$. For $N \sim 6$ we find that $m_\phi \sim H/6$ and therefore the slow-roll condition is satisfied and inflation happens. We see that the inflaton mass is very small $m_\phi \sim 2 \times 10^{-25}$ GeV as required. Thus, late D-term inflation satisfies the requirements of late inflation such as the very small Hubble constant and inflaton mass to solve the radion problem.

Note that during late D-term inflation the magnitude of density perturbations is very small

$$\frac{\delta\rho}{\rho} \sim \sqrt{\frac{32\pi}{45}} \frac{V^{3/2}}{V' M_P^3} \,. \tag{2.15}$$

Substituting the potential for D-term inflation from above we get $\delta\rho/\rho \sim 10^{-13}$ which is completely negligible. This is fine because the density perturbations of the early inflationary era were of the corect magnitude and (almost) scale invariant. Therefore, the magnitude of the density perturbations satisfies COBE constraints for the period of late inflation.

Finally as mentioned in [13] a very light inflaton as above cannot in general reheat the universe up to the MeV scale. However, note that after D-term inflation ends the inflaton mass becomes much larger than what it was during inflation. After D-term inflation

$$m_\phi^2 \sim \lambda^2 M^2 \sim 3 \times 10^9 \text{ GeV}^2 \,. \tag{2.16}$$

The main decay channel for the inflaton is to two fermionic σ's due to the large coupling coming from the tree level superpotential in eq. (2.7). The decay rate of the inflaton into two fermionic σ's is given by (note that the mass of σ after inflation vanishes since $\phi = 0$)

$$\Gamma \sim \frac{\lambda^2 m_\phi}{8\pi} \sim 2 \times 10^3 \text{ GeV} \,. \tag{2.17}$$

This is much larger than the Hubble constant, $H \sim 10^{-22}$ GeV and therefore all the vacuum energy will be converted into heat very efficiently. As a result,

$$T_R^4 \sim g^2 M^4 \tag{2.18}$$

and $T_R \sim \sqrt{g} M \sim 10^{-2}$ GeV. This is about the normalcy temperature required in order to reheat the brane and not predominantly the the bulk by graviton production [25]. It is also above the nucleosynthesis scale as required for maintaining the success of late stages of cosmology.

3. Conclusions

In this letter we have shown that late D-term inflation can solve the cosmological moduli problem which arises in inflationary models at the TeV scale. In order for this to be possible the Hubble constant and the inflaton mass must be extremely small. This requires the large dimensions to be stabilized before late D-term inflation. This is a reasonable assumption since the Hubble constant during late inflation is extremely small and therefore it happens at very late times. The size of the large compact dimensions is responsible for the very small coupling of the anomalous U(1) which in turn results in the extremely small Hubble constant and inflaton mass required. Even though the inflaton mass (during late inflation) is extremely small its decay into fermionic σ's can reheat the universe to the MeV scale because the inflaton mass after inflation is many orders of magnitude larger. This is one of the important properties of D-term inflation (or actually hybrid inflation).

The proposed solution of the cosmological moduli problem by late D-term inflation is more general than the above example. In particular, the modulus does not have to be the radion and the early phase of inflation does not have to be asymmetric inflation. If one assumes that the compact radii are fixed before early inflation then this can be realized by D-term inflation [17, 19]. Since D-term inflation is realized in the context of type I string (or type IIB orientifold) compactifications there will be string moduli such as the dilaton, untwisted and twisted moduli. (For the phenomenology of these see [26].) These generically are expected to have very small masses of $O(10^{-9}$ GeV) which they obtain after supersymmetry breaking around M_s on the brane. As usual, there will be a cosmological moduli problem associated with them after early D-term inflation with $H \sim 10^{-10}$ GeV ends and the universe is reheated. The amount of e-foldings required to solve this problem is exactly what was needed to solve the radion problem of asymmetric inflation, about 6–7 e-foldings. Once again we need late inflation with a very small Hubble constant at least smaller than the moduli masses. However, in the previous section we saw that for the radion a much stronger bound on H was obtained from the requirement that late inflation should not introduce the cosmological moduli problem. The bound was obtained from the detailed knowledge of the radion minimum during inflation. This is not in general possible for string moduli but it is safe to assume the same bound on H in this case too.

In this scenario there are two anomalous gauged U(1) symmetries with vastly different scales, e.g. $M_1 \sim 10^{10}$ GeV and $M_2 \sim 10^4$ GeV which refer to the scales of early and late D-term inflation. Equivalently, these two scales refer to the VEVs of two twisted moduli which parametrize blowups of different orbifold singularities. It is interesting to note that type I string orbifold models generically have more than one anomalous gauged U(1)'s [21, 23]. However we do not have an expalnation for the two vastly different scales.

Above we mainly concentrated on the case with two large compact dimensions. For more than two large dimensions the only difference is in the string scale. In these cases the bound on the string (or higher dimensional Planck) scale is weaker, i.e. $M_s > 1$ TeV. This increases the modulus (or radion) mass. In addition, the bound on the Hubble constant during late inflation becomes much weaker, e.g. from $O(10^{-19}$ GeV) up to $O(10^{-14}$ GeV) depending on the number of large dimensions. In any case, it is clear that all our results apply equally well to the cases with more than two large dimensions.

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